IEEE Access
Multidisciplinary : Rapid Review : Open Access Journal

Received February 24, 2020, accepted March 17, 2020, date of publication April 1, 2020, date of current version April 14, 2020.

Digital Object Identifier 10.1109/ACCESS.2020.2984744

An EKF-Based Performance Enhancement Scheme for Stochastic Nonlinear Systems by Dynamic Set-Point Adjustment

XIAFEI TANG [1], QICHUN ZHANG [2], (Senior Member, IEEE), AND LIANG HU [3], (Member, IEEE)

[1]Engineering Research Centre of the Ministry of Education Power Grid Security Monitoring and Control Technology, Changsha University of Science and Technology, Changsha 410114, China
[2]Department of Computer Science, University of Bradford, Bradford BD7 1DP, U.K.
[3]School of Computer Science and Electronic Engineering, University of Essex, Colchester CO4 3SQ, U.K.

Corresponding author: Qichun Zhang (q.zhang17@bradford.ac.uk)

ABSTRACT In this paper, a performance enhancement scheme has been investigated for a class of stochastic nonlinear systems via set-point adjustment. Considering the practical industrial processes, the multi-layer systematic structure has been adopted to achieve the control design requirements subjected to random noise. The basic loop control is given by PID design while the parameters have been fixed after the design phase. Alternatively, we can consider that there exists an unadjustable loop control. Then, the additional loop is designed for performance enhancement in terms of the tracking accuracy. In particular, a novel approach has been presented to dynamically adjust the set-points using the estimated states of the systems through extended Kalman filter (EKF). Minimising the entropy criterion, the parameters of the set-point adjustment controller can be optimised which will enhance the performance of the entire closed-loop systems. Based upon the presented scheme, the stochastic stability analysis has been given to demonstrate that the closed-loop tracking errors are bounded in probability one. To indicate the effectiveness of the presented control scheme, the numerical examples have been given and the simulation results imply that the designed systems are bounded and the tracking performance can be enhanced simultaneously. In summary, a new framework for system performance enhancement has been presented even if the loop control is unadjustable which forms the main contribution of this paper.

INDEX TERMS Stochastic nonlinear systems, EKF, performance enhancement, set-point adjustment, double-layer control structure, existing control loop, operational optimisation.

I. INTRODUCTION

The system structures of modern industrial processes are mostly complex with various interconnections which follow the developments of the various performance requirements. For practical control application, PID controller is widely used because of its simple structure [1]. In particular, PID controller only has 3 parameters to turning which is rapid and convenient as a benefit to the process operators, meanwhile it shortens the training time for new operators. In this case, most of the process control loop use PID to achieve the basic control objective. However, the performance of the systems with standard PID controllers cannot meet the design requirements because of the nonlinear dynamics, strong external disturbance and the random noise.

As aforementioned above, the industrial processes are with parameter-fixed PID controllers. For example, a lot of controllers are implemented using analog circuits which are integrated into the actuators as fix packages which mean that the design parameters of the existing control loops are difficult to adjust after the design phase [2]. It is difficult to enhance the performance of the systems with such existing control loops as mostly of the performance enhancement approaches are based on parametric optimisation. Thus, performance enhancement problem is hardly solved with the single existing control loop. In other words, it is important to develop a new control scheme to enhance the system performance

The associate editor coordinating the review of this manuscript and approving it for publication was Zheng H. Zhu.

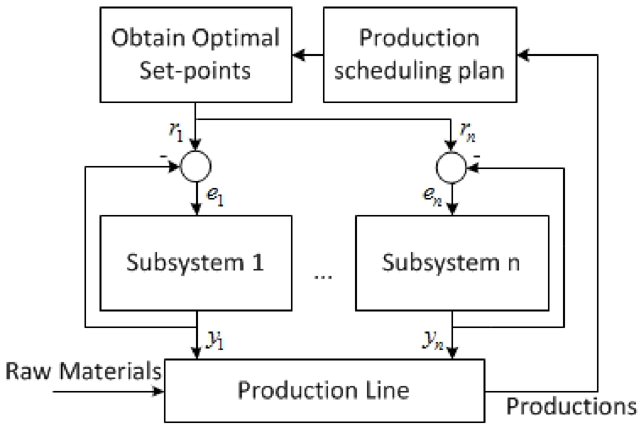


FIGURE 1. The schematic diagram for the operational optimal control scheme .

Motivated by the operational control [3], [4] whose systematic diagram is given by Fig.(1), an additional performance enhancement loop can be designed onto the existing control loop which is inspired by the cascade control [5]. Using this idea, even if the existing control loop e.g. PID controller cannot track the system reference perfectly regarding to the uncertainties, the performance enhancement loop will re-adjust the dynamic set-point of the existing control loop to compensate the residuals. In summary, the purpose of this paper is to design a performance enhancement loop with optimal set-point adjustment and also to establish a scheme to solve the convergence problem.

It is ideal to design this additional performance enhancement loop using system full states which reflect all the internal information of the system dynamics. However, the real system states are mostly unmeasurable which means it is difficult to design a full state-based approach. To estimate the states, the filtering methods have been well-developed based upon the famous Kalman filter [6] considering the Gaussian noise. For nonlinear stochastic systems, the extended Kalman filter (EKF) was presented [7]. Compared to EKF, unscented Kalman filter (UKF) [8], [9] gives a solution to enhance the accuracy of filtering. Currently, particle filter (PF) [10], entropy filter [11] and non-fragile H_∞ robust filter get extensive attentions from the view of filtering theory [12]. However, these filters take huge amount of calculation while robust filter design need to solve a Riccati equation which restricts its application. Therefore, similar to the PID control strategy, EKF is the most common filter design method for applications.

This paper presents a new control scheme to enhance the performance of the systems in terms of randomness attenuation using estimated state of the systems. Two-layer structure is presented to guarantee that the existing loop will never be changed once the parameters are selected. Then, the convergence of the entire closed-loop system is given in probability one. Extended our previous result [13], the potential framework has been obtained. Using this framework, the tracking performance of stochastic system outputs has been optimised where the kernel density estimation (KDE) [14] is used to approximate the entropy of the system outputs. In addition, the parametric optimisation has been achieved by constrained convex optimisation approach. Generally, the presented scheme can be extended simply and it is shown that the presented control scheme can be fulfilled for real industrial processes.

The rest parts of the paper are organised as follows: some preliminaries including problem description are introduced in Section II. Section III presents the control scheme via set-point adjustment. The stability analysis and parameter optimisation are given in Section IV and V, respectively. The numerical examples are shown in Section VI and the conclusion of the paper has been given by Section VII.

II. FORMULATION

A. PROBLEM DESCRIPTION

In this paper, we consider the following general multivariable discrete-time system model which is subjected to additive Gaussian white noise. Particularly, the i-th subsystem model is described as follows:

$$x_{i,k+1} = f_i\left(x_{i,k}, u_{i,k}\right) + G_i w_{i,k} \tag{1a}$$

$$y_{i,k} = C_i x_{i,k} + D_{i,k} v_{i,k} \tag{1b}$$

where i stands for the index of the subsystems, $1 \leq i \leq N$. $x_i \in R^{n_i}$ is the subsystem state vector, $y_i \in R^{m_i}$ is the subsystem output, $u_i \in R^{s_i}$ is the subsystem control input. Moreover, process noise $w_i \in R^{p_i}$ and measurement noise $v_i \in R^{q_i}$ are both Gaussian. Real positive integers n_i, m_i, s_i, p_i and q_i denote the dimension of the i-th subsystem vectors. In addition, real matrices G_i, C_i and D_i stand for the system matrices with appropriate dimensions. Meanwhile, $f_i : R^{n_i} \times R^{s_i} \to R^{n_i}$ are real general nonlinear functions. Note that the nonlinearities of the subsystem will result in the non-Gaussian distributions for both subsystem state and subsystem output.

For practical application, most of the controllers are developed using linear model. Thus, the linearised model with known equilibrium for the subsystem can be obtained as follows:

$$x_{i,k+1} = A_i x_{i,k} + B_i u_{i,k} + g_{i,k}\left(x_{i,k}, u_{i,k}\right) + G_i w_{i,k} \tag{2a}$$

$$y_{i,k} = C_i x_{i,k} + D_{i,k} v_{i,k} \tag{2b}$$

where real matrices A_i, B_i, C_i and D_i are system coefficient matrices following the linearisation operation regarding to the i-th subsystem. The un-modelled dynamics can be reflected by the unknown nonlinear function $g_i : R^{n_i} \times R^{s_i} \to R^{n_i}$. In particular, we can further denote the equilibrium as $\left(x_i^*, u_i^*\right)$, then the coefficient matrices can be obtained by the following equation.

$$\{A_i, B_i\} = \left\{\frac{\partial f_i\left(x_i, u_i\right)}{\partial x_i}, \frac{\partial f_i\left(x_i, u_i\right)}{\partial u_i}\right\}\Bigg|_{x_i = x_i^*, u_i = u_i^*} \tag{3}$$

Without loss of the generality, the composite format of the system model can be used through the entire manuscript as combining the subsystems together which is simplified the system formulation.

$$x_{k+1} = Ax_k + Bu_k + g_k(x_k, u_k) + Gw_k \quad (4a)$$
$$y = Cx_k + Dv_k \quad (4b)$$

where $x = [x_1^T, \ldots, x_N^T]^T$, $y = [y_1^T, \ldots, y_N^T]^T$, $u = [u_1^T, \ldots, u_N^T]^T$, $w = [w_1^T, \ldots, w_N^T]^T$, $v = [v_1^T, \ldots, v_N^T]^T$, $A = diag\{A_1, \ldots, A_N\}$, $B = diag\{B_1, \ldots, B_N\}$, $C = diag\{C_1, \ldots, C_N\}$, $G = diag\{G_1, \ldots, G_N\}$, $D = diag\{D_1, \ldots, D_N\}$. Note that the un-modelled dynamics described by nonlinear functions are also composited into compact format as $g(x, u) = [g_1^T(x_1, u_1), \ldots, g_N^T(x_N, u_N)]^T$.

Remark 1: For each subsystem model (7), the output equation can also be generalised to a nonlinear function as the state equation. Due to the fact that most of the sensors for real industrial processes are linear, the output equations are linear in this paper. In other words, the working range of the sensors presents a linear property.

Remark 2: In model (2), the unknown nonlinear function term $g_{i,k}(x_{i,k}, u_{i,k})$ represents the un-modelled dynamics of the subsystems. It means that the nonlinear function $f_i(x_{i,k}, u_{i,k})$ in model (7) can even be partly unknown, which can also guarantee that these two models are equivalent for each time instant k.

The basic control objective is to design a controller such that the system output y_k can perfectly track the system reference y_k^*. To achieve the objective, the basic loop controller can be designed using the linear part of the system model. In particular, the loop tracking error $e_{\bar{k}}$ can be defined. However, it is difficult to achieve the precise tracking due to the nonlinear un-modelled dynamics. Thus, we can consider to adjust the dynamic set-point for the basic loop $r_{\bar{k}}$. Basically, the residual ε_k can be further defined.

$$e_{\bar{k}} = r_{\bar{k}} - y_{\bar{k}}, \quad \varepsilon_k = y_k^* - y_k \quad (5)$$

Note that inner loop should have higher sampling speed which means $\bar{k}$ as a sampling instant is faster than k. In this case, the system has been modelled by two different time-scales however both of them reflect the same system dynamic properties. Thus, $y_{\bar{k}}$ and y_k are equivalent from the view of multiple time scales. Meanwhile, the coefficient matrices should be specified with the sampling index which are denoted as $A_{\bar{k}}$, $B_{\bar{k}}$, $C_{\bar{k}}$, $D_{\bar{k}}$ and $G_{\bar{k}}$.

Remark 3: In this paper, y_k^* is the reference signal which should be tracked by the system output y_k. On the other hand, r_k denotes the set-point of the control loop which might be different from y_k^*. For single layer control strategies, the reference signal y_k^* is equal to the set-point r_k.

The loop controller can be designed by $e_{\bar{k}}$ and the parameters are fixed. Compared to classical process control, the set-points of the subsystems are not equal to the reference signals. In this paper, the set-points have been designed as the dynamic signals to enhance the performance of the closed-loop systems. In particular, the control inputs and the dynamics set-points can be presented as general real nonlinear functions.

$$u_{\bar{k}} = f_u\left(\bar{e}_{\bar{k}}, \quad \bar{u}_{\bar{k}-1}\right), r_k = f_r\left(\bar{\varepsilon}_k, \bar{\hat{x}}_k, \bar{r}_{k-1}\right) \quad (6)$$

where $\hat{x}$ denotes the estimated system state. In addition, we have

$$\bar{e}_k = [e_k, \ldots, e_0] \quad (7a)$$
$$\bar{u}_{k-1} = [u_{k-1}, \ldots, u_0] \quad (7b)$$
$$\bar{\varepsilon}_k = [\varepsilon_k, \ldots, \varepsilon_0] \quad (7c)$$
$$\bar{\hat{x}}_k = [\hat{x}_k, \ldots, \hat{x}_0] \quad (7d)$$
$$\bar{r}_{k-1} = [r_{k-1}, \ldots, r_0] \quad (7e)$$

The performance of the closed-loop systems can be enhanced in many respects, such as tracking performance [15], probabilistic decoupling [16], [17], etc. The general performance criterion can be formulated as

$$J_k = \sum_{i=1}^{k} \varphi\left(y_k, y_k^*, r_k, u_k, x_k, \hat{x}_k\right) \quad (8)$$

where $\varphi(\cdot)$ denotes a general function with continuous first and second partial derivatives. Basically, in order to formula the performance criterion for stochastic processes, the statistical operations are usually used, for example, the function φ would be expectation function or probability density function.

From the view of generalised control scheme, the specific objective of this paper is to obtain a function f_r which can minimise the performance criterion (8) with a un-adjustable f_u. In particular, the performance criterion (8) can be further expressed by Eq. (59) as minimum entropy which represents the performance enhancement of system output tracking.

Remark 4: For most of industrial processes, the states cannot be measured directly. However, it is easy to collect the input and the output data of the subsystems. Based on filtering theory, the estimated states can be obtained to design the dynamic set-point signals which implies that the closed-loop can get more information to use not only the system outputs. Note that the existing control loops are without any change which is significant in industrial sites.

B. THE EXTENDED KALMAN FILTER

Using the system model (4), the EKF design [18] can be adopted in this paper.

Definition 5: The extended Kalman filter can be defined using the equations below in discrete-time:

- State estimation:

$$\hat{x}_{k+1} = f\left(\hat{x}_k, u_k\right) + K_{f,k}\left(y_k - C\hat{x}_k\right) \quad (9a)$$

- Kalman gain:

$$K_{f,k} = AP_{f,k}C^T\left(CP_{f,k}C^T + R_k\right)^{-1} \quad (9b)$$

- Riccati difference equation:

$$P_{f,k+1} = AP_{f,k}A^T + Q_k - AP_{f,k}C^TK_{f,k}^T \quad (9c)$$

where positive-definite matrices Q_k and R_k are symmetric and time-varying.

Remark 6: The linearization is covered by the modeling procedure. Q_k and R_k are chosen as the covariance matrices of random noises w_k and v_k normally. Especially, if the EKF is used as an observer for a deterministic system, Q_k and R_k are equal to 0.

C. STOCHASTIC BOUNDEDNESS

Once the system states are approximated by EKF, the estimation error is formulated as follows:

$$\tilde{x}_k = x_k - \hat{x}_k \tag{10}$$

The error vector in terms of the closed-loop system should be generalised as follows:

$$\zeta_k = \left[\tilde{x}_k^T, \varepsilon_k^T\right]^T \tag{11}$$

Then, the following Definitions and Lemma are recalled in order to analyse the dynamics of the error ζ_k.

Definition 7: The stochastic process ζ_k is said to be exponentially bounded in mean square sense, if there exist real numbers $\eta, \upsilon > 0$ and $0 < \vartheta < 1$ such that for $\forall k$, the following inequality holds.

$$E\left\{\|\zeta_k\|^2\right\} \leq \eta\|\zeta_0\|^2\vartheta^k + \upsilon \tag{12}$$

where $E\{\cdot\}$ stands for mathematical expectation and and $\|\cdot\|$ denotes norm operation.

Definition 8: The stochastic process ζ_k is said to be bounded with probability one, if the following equation holds.

$$\Pr\left\{\limsup_{k\to\infty}\|\zeta_k\| < \infty\right\} = 1 \tag{13}$$

where $\Pr\{\cdot\}$ denotes the probability value.

Lemma 9: For stochastic process ζ_k, assume there is a stochastic process $V_k(\zeta_k)$ as well as real positive numbers $\bar{v}, \underline{v}, \mu, \alpha, \beta > 0$ and $0 < \alpha + \beta \leq \underline{v}$, such that

$$\underline{v}\|\zeta_k\|^2 \leq V_k(\zeta_k) \leq \bar{v}\|\zeta_k\|^2 \tag{14}$$

and

$$E\left\{V_{k+1}(\zeta_{k+1})|_{\zeta_k}\right\} \leq \alpha\|\zeta_k\|^2 + \beta\|\zeta_k\| + \mu \tag{15}$$

are fulfilled for every solution of stochastic process ζ_k. Then ζ_k is bounded with probability one. Moreover it is also exponentially bounded in mean square sense, which implies that for $\forall k \geq 0$, we have

$$E\left\{\|\zeta_k\|^2\right\} \leq \frac{\bar{v}}{\underline{v}}E\left\{\|\zeta_0\|^2\right\}\left(\frac{\alpha+\beta}{\underline{v}}\right)^k + \frac{\beta+\mu}{\underline{v}-\alpha-\beta} \tag{16}$$

Proof of Lemma 9: Since all the three terms of the right side of the inequality (16) are positive, if $\|\zeta_k\| > 1$ we have

$$E\left\{V_{k+1}(\zeta_{k+1})|_{\zeta_k}\right\} \leq (\alpha+\beta)\|\zeta_k\|^2 + \mu \tag{17}$$

otherwise, if $0 \leq \|\zeta_k\| \leq 1$,

$$E\left\{V_{k+1}(\zeta_{k+1})|_{\zeta_k}\right\} \leq \alpha\|\zeta_k\|^2 + \beta + \mu \tag{18}$$

Therefore, we can claim that

$$E\left\{V_{k+1}(\zeta_{k+1})|_{\zeta_k}\right\} \leq (\alpha+\beta)\|\zeta_k\|^2 + \beta + \mu \tag{19}$$

Inequality (14) leads to

$$\|\zeta_k\|^2 \leq \frac{1}{\underline{v}}V_k(\zeta_k) \tag{20}$$

Substituting inequality (20) to inequality (19), we can obtain

$$E\left\{V_{k+1}(\zeta_{k+1})|_{\zeta_k}\right\} \leq \frac{\alpha+\beta}{\underline{v}}V_k(\zeta_k) + \beta + \mu \tag{21}$$

Combining Theorem 4.1 in [19], Theorem 2 in [20] and Lemma 2.1 in [21], the inequality (16) can be obtained using inequality (21) and the proof is completed. ∎

D. KERNEL DENSITY ESTIMATION

For most of the stochastic processes with the performance criterion which is formulated by Eq.(8), function φ would be a function of probability density functions (PDF). The kernel density estimation (KDE) [14] is a well-known data-based approach to estimate the PDFs of the random variables.

For a continuous random variable $x \in \mathbb{R}^n$, with its sampled data points $\{x_i : i = 1, \ldots, N\}$, the probability density function of can be estimated using Parzen window method [22] to give

$$\hat{\gamma}(x) = \frac{1}{N}\sum_{i=1}^{N} G_\Sigma(x - x_i) \tag{22}$$

where $G_\Sigma(\cdot)$ is the Gaussian function defined as follows:

$$G_\Sigma(x) = (2\pi)^{-\frac{n}{2}}(\det\Sigma)^{-\frac{1}{2}}\exp\left(-\frac{1}{2}x^T\Sigma^{-1}x\right) \tag{23}$$

Therefore, almost all of the concepts based on probability density functions, such as entropy, mutual information, etc, can be estimated by date-based approximation using the approach we introduced above [23].

E. ASSUMPTIONS AND NOTATIONS

Some assumptions are given for the investigated system as follows:

H1: The investigated system (4) is controllable and observable.

H2: Suppose the matrix CB is non-singular matrix.

H3: There exist real positive numbers L_1 and L_2, such that the following Lipschitz conditions hold.

$$\left\|g_k(x_k, u_k) - g_k\left(\hat{x}_k, u_k\right)\right\| \leq L_1\|\tilde{x}_k\| \tag{24a}$$
$$\|g_k(x_k, u_k)\| \leq L_2\|x_k\| \tag{24b}$$

H4: There exist four positive real numbers $\bar{a}$, $\bar{b}$, $\bar{c}$ and $\bar{d}$ such that the following inequalities hold.

$$\|x_k\| \leq \bar{a}\|\varepsilon_k\| + \bar{b} \tag{25a}$$
$$\|\tilde{x}_{k-1}\| \leq \bar{c}\|\tilde{x}_k\| + \bar{d} \tag{25b}$$

Remark 10: From the view of the control applications, all these assumptions are not difficult to satisfy. While the standard Lipschitz condition is widely used in control theory for the stability analysis of the closed-loop systems.

III. CONTROL SCHEME BY SET-POINT ADJUSTMENT

Based upon the materials in Section I and II, the double-layer control scheme is designed in this section. For subsystem loops, the discrete-time PID design is adopted. Moreover, the dynamic set-point can be achieved based upon the estimated system information. To clarify the structure of the designed system, Fig. 2 shows the schematic diagram.

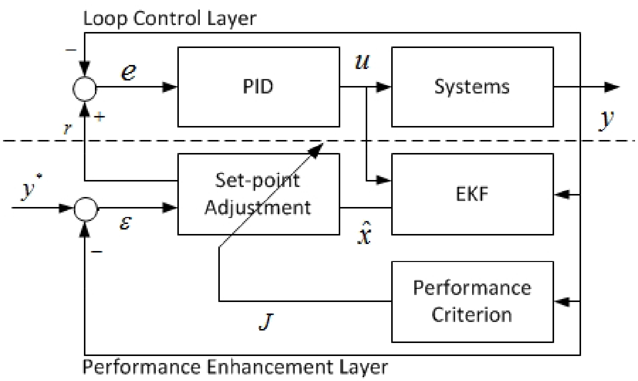


FIGURE 2. **The schematic diagram for the control scheme .**

A. LOOP CONTROL LAYER DESIGN

We define $K = [K_P, K_I, K_D]$ and $z_{\bar{k}} = \sum_{i=0}^{k} e_{\bar{k}}$, thus the PID design can be implemented using the following equations.

$$u_{\bar{k}} = K\left[e_{\bar{k}}^T, z_{\bar{k}}^T, e_{\bar{k}}^T - e_{\bar{k}-1}^T\right]^T \quad (26a)$$

$$z_{\bar{k}} = z_{\bar{k}-1} + e_{\bar{k}} \quad (26b)$$

where matrix K stands for the parameter of the PID controller.

Further defining $\theta_{\bar{k}} = \left[x_{\bar{k}}^T, z_{\bar{k}}^T, x_{\bar{k}-1}^T\right]^T$ as a new vector-valued variable, then the following model can be obtained to represent the dynamics of $\theta_{\bar{k}}$.

$$\theta_{k+1} = A_d\theta_k + B_d r_k + G_d w_k + E_d g_k(x_k, u_k) \quad (27a)$$

$$y_k = C_d\theta_k + D_d v_k \quad (27b)$$

where $E_d = \left[\bar{1}, 0, 0\right]^T$. Note that both 0 and $\bar{1}$ here are vectors. Furthermore, the coefficient matrices are formulated as $A_d = \bar{A} + \bar{B}K\bar{C}$, $B_d = \left[B_{\bar{k}}K_P, I, 0\right]^T$, $C_d = \left[C_{\bar{k}}, 0, 0\right]$, $D_d = \left[D_{\bar{k}}, 0, 0\right]^T$. $G_d = \left[G_{\bar{k}}, 0, 0\right]^T$, while $\bar{B} = \left[B_{\bar{k}}, 0, 0\right]^T$,

$$\bar{A} = \begin{bmatrix} A_{\bar{k}} & 0 & 0 \\ -C_{\bar{k}} & I & 0 \\ I & 0 & 0 \end{bmatrix}, \quad \bar{C} = \begin{bmatrix} -C_{\bar{k}} & 0 & 0 \\ 0 & I & 0 \\ -C_{\bar{k}} & 0 & C_{\bar{k}} \end{bmatrix} \quad (28)$$

Ideally, the parameters can be obtained ignoring the unknown nonlinear high-order term, which results in the following simplified model.

$$\theta_{\bar{k}+1} = A_d\theta_{\bar{k}} + \bar{d}_{1,\bar{k}} \quad (29a)$$

$$y_{\bar{k}} = C_d\theta_{\bar{k}} + \bar{d}_{2,\bar{k}} \quad (29b)$$

where $\bar{d}_{1,\bar{k}} = B_d y_{\bar{k}}^* + G_d w_{\bar{k}}$ and $\bar{d}_{2,\bar{k}} = D_d v_{\bar{k}}$ are treated as the external disturbances.

Similar to the approach in [24] regarding to the parametric selection, a proposition is given as follows:

Proposition 11: The investigated system (27) is said to be stable in ideal situation (28). If the parameter matrix $K = W^{-1}Y$ has been obtained from the following linear matrix inequality (LMI):

$$\begin{bmatrix} -M & M\bar{A} + \bar{B}Y\bar{C} \\ \left(M\bar{A} + \bar{B}YC\right)^T & -(1-\bar{\alpha})M \end{bmatrix} < 0 \quad (30)$$

where $\bar{\alpha} \geq 0$ stands for the decay rate. M is a symmetric positive-definite matrix and $Y = WK$. In addition, $M\bar{B} = \bar{B}W$.

Proof of Proposition 11: Due to the similarity of the proof in [24], the proposition proof has been omited in this paper. ■

B. PERFORMANCE ENHANCEMENT LAYER DESIGN

In practice, the system cannot work in the ideal case. As aforementioned, the parameters of the PID controller are fixed and un-adjustable after the design phase. In this case, we have to consider the PID loops as the existing loops, where only the set-points are dynamically adjustable with a slower sampling rate. For the outer loop design, the vector-valued set-point signal is designed using the following approach.

$$r_k = (C_dB_d)^{-1}\left(y_{k+1}^* - C_dA_d\hat{\theta}_k - \Theta_1\varepsilon_k - \Theta_2\Lambda_k\right) \quad (31a)$$

$$\Lambda_k = \kappa_1\Lambda_{k-1} + \kappa_2\varepsilon_{k-1} \quad (31b)$$

where Θ_1 and Θ_2 are the design parameters and $\hat{\theta}_k = \left[\hat{x}_k^T, z_k^T, \hat{x}_{k-1}^T\right]^T$ is the vector-valued estimation for θ_k. κ_i denotes the real positive number where $0 < \kappa_i < 1$. Based upon the assumption (H2), the matrix C_dB_d is nonsingular which implies that the adjustment scheme is implementable.

In the next two sections, the analysis are given for convergence and the optimisation of design parameter Θ, respectively.

Remark 12: For the most of the control applications to industrial processes, the reference signals are real constants since the operational points are fixed. Moreover, if the reference signals are time-varying, for example the operational control, the optimal reference signals are predicable.

Remark 13: Notice that the set-point of the controlled system should have a lower sampling rate due to the dynamic of the loop control layer which needs extra time to response the reset of the system reference signal. We can use the lower sampling time to discretise the system, then the set-point can be obtained based on the invariance of the discrete-time system based upon various sampling times. To simplify the mathematics formulas, Eq.(31) is designed by the same sampling rate as the loop control layer, however the following analysis and design will not be affected even if the sampling rate has been slowed down.

Remark 14: As the set-points have been designed with slow sampling time, the discrete-time set-point update can be considered as a signal switching strategy where the switching rude are dynamically obtained by the estimated system states following the performance criterion.

IV. CONVERGENCE ANALYSIS IN PROBABILITY SENSE

Firstly, we look at the dynamics of the estimation error of the system states, which are given as follows:

$$\tilde{x}_{k+1} = \left(A - K_f C\right)\tilde{x}_k + Gw_k - K_f Dv_k + g_k\left(x_k, u_k\right) - g_k\left(\hat{x}_k, u_k\right) \quad (32)$$

Moreover, the dynamics of the residual can be described as

$$\varepsilon_{k+1} = y^*_{k+1} - C_d A_d \theta_k - C_d B_d r_k - C_d G_d w_k - D_d v_k - C_d E_d g_k\left(x_k, u_k\right) \quad (33)$$

Since the term $C_d A_d \theta_k$ can be rewritten by

$$C_d A_d \theta_k = \Pi_1 x_k + \Pi_2 z_k + \Pi_3 x_{k-1} \quad (34)$$

In particular, $\Pi_1 = -C\left(BCK_D - A + BCK_P\right)$, $\Pi_2 = CBK_I$ and $\Pi_3 = CBCK_D$.

Substituting the set-point design (31) and Eq. (34) to Eq. (33), we have

$$\varepsilon_{k+1} = \Theta_1 \varepsilon_k + \Theta_2 \Lambda_k - \Pi_1 \tilde{x}_k - \Pi_3 \tilde{x}_{k-1} - C_d G_d w_k - D_d v_k - C_d E_d g_k\left(x_k, u_k\right) \quad (35)$$

Therefore, the generalised error dynamics of the investigated system can be described as follows:

$$\zeta_{k+1} = A_f \zeta_k + d_k + s_k \quad (36)$$

where $\zeta_k = \left[\tilde{x}_k^T, \varepsilon_k^T, \Lambda_k^T\right]^T$ and

$$A_f = \begin{bmatrix} A - K_f C & 0 & 0 \\ -\Pi_1 & \Theta_1 & \Theta_2 \\ 0 & \kappa_2 & \kappa_1 \end{bmatrix} \quad (37a)$$

$$d_k = \begin{bmatrix} Gw_k - K_f Dv_k \\ -C_d G_d w_k - D_d v_k \\ 0 \end{bmatrix} \quad (37b)$$

$$s_k = \begin{bmatrix} g_k\left(x_k, u_k\right) - g_k\left(\hat{x}_k, u_k\right) \\ -\Pi_3 \tilde{x}_{k-1} - C_d E_d g_k\left(x_k, u_k\right) \\ 0 \end{bmatrix} \quad (37c)$$

Based on Eq. (36), a theorem has been summarised as the analytical result of this Section.

Theorem 15: The stochastic systems (7) with the control inputs (26) and dynamic set-points (31), if the following conditions are satisfied, then the closed-loop systems are stable with probability one. Moreover, the outputs of the systems are stable in mean square sense.

- The assumptions (24,25) for the stochastic systems hold.
- Designing the parameters K, K_f and Θ such that

$$0 < \left\|A_f\right\| \leq \frac{1 - \alpha - L_1^2 - \bar{\delta}_1}{2\left(L_1 + \bar{\delta}_2\right)} < 1 \quad (38)$$

where

$$\bar{\delta}_1 = \bar{c}\left(\bar{c} + 2\bar{d}\right)\left\|\Pi_3\right\|^2 + 2\left(\bar{c} + \bar{d}\right)L_1\left\|\Pi_3\right\| + \left(\bar{a}+2\bar{b}\right)L_2^2\left\|C_d E_d\right\|^2 + 2\left(\bar{a} + \bar{b}\right)L_1 L_2\left\|C_d E_d\right\| + 2\left(\bar{a}\bar{c} + \bar{b}\bar{c} + \bar{a}\bar{d}\right)L_2\left\|\Pi_3\right\|\left\|C_d E_d\right\| \quad (39a)$$

$$\bar{\delta}_2 = \left(\bar{c} + \bar{d}\right)\left\|\Pi_3\right\| + \left(\bar{a} + \bar{b}\right)L_2\left\|C_d E_d\right\| \quad (39b)$$

To proof this theorem, we develop some useful lemmas as follows:

Lemma 16: Suppose A is a real non-singular matrix, there exist a real positive definite matrix P and a real constant $\alpha > 0$ such that

$$A^T P^{-1} A \leq \alpha P^{-1} \quad (40)$$

Moreover, if $\|A\| < 1$, the constant $0 < \alpha < 1$ exists.

Lemma 17: Suppose P is a real positive definite matrix, $\underline{p}$ is the infimum of P. Based on the error dynamic (36), the following inequalities holds.

$$s_k^T P^{-1}\left(2A_f \zeta_k + s_k\right) \leq \frac{1}{\underline{p}}\left(N_1\left\|\zeta_k\right\|^2 + N_2\left\|\zeta_k\right\| + M_3^2\right) \quad (41a)$$

$$E\left\{d_k^T P^{-1} d_k\right\} \leq \frac{\delta^2}{\underline{p}} \quad (41b)$$

where

$$N_1 = \left(M_1 + M_2\right)\left(2\left\|A_f\right\| + M_1 + M_2\right) \quad (42a)$$

$$N_2 = 2M_3\left(\left\|A_f\right\| + M_1 + M_2\right) \quad (42b)$$

$$\delta = \left(\|G\| + \left\|C_d G_d\right\|\right)E\left\{\left\|w_k\right\|\right\} + \left(\left\|K_f D\right\| + \left\|D_d\right\|\right)E\left\{\left\|v_k\right\|\right\} \quad (42c)$$

while

$$M_1 = L_1 + \bar{c}\left\|\Pi_3\right\| \quad (43a)$$

$$M_2 = \bar{a}L_2\left\|C_d E_d\right\| \quad (43b)$$

$$M_3 = \bar{d}\left\|\Pi_3\right\| + \bar{b}L_2\left\|C_d E_d\right\| \quad (43c)$$

The proofs of these lemmas are given in the Appendix A. Using these lemmas, the proof of theorem 15 is obtained as follows:

Proof of Theorem 15: Lemma 16 results in a positive-definite matrix P and then the following Lyapunov function candidate can be obtained.

$$V_{k+1}\left(\zeta_{k+1}\right) = \zeta_{k+1}^T P^{-1} \zeta_{k+1} \quad (44)$$

Thus,

$$V_{k+1}\left(\zeta_{k+1}\right) \quad (45)$$
$$= \left(A_f \zeta_k + d_k + s_k\right)^T P^{-1}\left(A_f \zeta_k + d_k + s_k\right)$$
$$= \zeta_k^T A_f^T P^{-1} A_f \zeta_k + 2d_k^T P^{-1}\left(A_f \zeta_k + s_k\right) + s_k^T P^{-1}\left(2A_f \zeta_k + s_k\right) + d_k^T P^{-1} d_k \quad (46)$$

Taking the conditional expectation, it shows that $E\{d_k^T P^{-1}\left(A_f\zeta_k + s_k\right)\big|_{\zeta_k}\}$ vanishes. Lemma 16 and Lemma 17 lead to the following inequality.

$$\begin{aligned}
&E\left\{V_{k+1}\left(\zeta_{k+1}\right)\big|_{\zeta_k}\right\} \\
&\quad\leq \alpha V_k\left(\zeta_k\right) + \frac{1}{\underline{p}}\left(N_1\|\zeta_k\|^2 + N_2\|\zeta_k\| + M_3^2\right) + \frac{\delta^2}{\underline{p}} \\
&\quad\leq \frac{\alpha}{\underline{p}}\|\zeta_k\|^2 + \frac{1}{\underline{p}}\left(N_1\|\zeta_k\|^2 + N_2\|\zeta_k\| + M_3^2\right) + \frac{\delta^2}{\underline{p}} \\
&\quad\leq \frac{\alpha + N_1}{\underline{p}}\|\zeta_k\|^2 + \frac{N_2}{\underline{p}}\|\zeta_k\| + \frac{M_3^2 + \delta^2}{\underline{p}}
\end{aligned} \tag{47}$$

where $\underline{p}$ is the infimum of P.

Using the condition of Lemma 9, the system error (36) is claimed as bounded with probability one when the inequality below always holds.

$$\frac{\alpha + N_1}{\underline{p}} + \frac{N_2}{\underline{p}} \leq \frac{1}{\bar{p}} \tag{48}$$

where $\bar{p}$ is the supremum of P.

To simplify the expression, we can represent the inequality as follows:

$$\alpha + N_1 + N_2 \leq 1 \tag{49}$$

Then, substituting Eq.(42) and Eq.(43), inequality (48) results in (38) which ends the proof. ■

V. PERFORMANCE ENHANCEMENT BY PARAMETRIC OPTIMISATION

The parametric optimisation becomes a constrained optimisation problem as the design parameters Θ_1 and Θ_2 have to be searched in a stable set as a result of Theorem 15. Considering A_f and the condition (38), the constraint for the optimisation problem can be formulated as

$$\|\Theta_1 + \Theta_2\| \leq \Theta_0 \tag{50}$$

which is a convex function and

$$\Theta_0 = \frac{1-\alpha-L_1^2-\bar{\delta}_1}{2\left(L_1+\bar{\delta}_2\right)} - \left\|A-K_f C\right\| - \|\Pi_1\| - \|\kappa_1\| - \|\kappa_2\| \tag{51}$$

It has been shown that this optimisation problem can be solved by convex optimisation approaches if the objective function is also a convex function. Therefore, in this section, the convexity analysis is given for performance enhancement criterion in terms of randomness attenuation. In particular, the minimum entropy criterion has been taken into account.

A. THE PERFORMANCE ENHANCEMENT FOR TRACKING PROPERTY

Motivated by [25], to enhance the performance of tracking property, the minimum quadratic Rényi entropy criterion for the residual can be used.

$$H_{2,k}\left(\varepsilon\right) = -\log\int_{\Omega}\gamma_{J,k}^2\left(\varepsilon\right)d\varepsilon \tag{52}$$

where $\gamma_J\left(\cdot\right)$ denotes the joint probability density function (JPDF) and Ω is the sampling set for random variable ε.

Since the JPDF can be estimated by kernel density estimation which is presented in Section II, Eq.(52) can be rewritten as follows:

$$H_{2,k}\left(\varepsilon\right) = -\log V_k\left(\varepsilon\right) \tag{53}$$

where V_k stands for information potentials [26]. Furthermore, it can be formulated by

$$V_k\left(\varepsilon\right) = \frac{1}{N^2}\sum_{i,j=1}^{N} G_{\sqrt{2}\Sigma}\left(\varepsilon_{i,k} - \varepsilon_{j,k}\right) \tag{54}$$

Then minimising of the entropy is equivalent to maximising the information potential because of the monotonic increasing property of the $\log\left(\cdot\right)$function and the following theorem is given.

Theorem 18: For the proposed control algorithm, there exist two real positive numbers $\delta_0 > 0$ and $\delta_k > 0$, such that the information potential is globally concave w.r.t. the design parameter Θ_1 and Θ_2 for all $\lambda_{\min}\left(\Sigma\right) > \delta_0$ and $k > \delta_k$. Thus the equivalent minimum entropy problem (52) is convex and has a global optimum.

Proof of Theorem 18: Denote $\varepsilon_{ij,k} = \varepsilon_{i,k} - \varepsilon_{j,k}$, then we have

$$\begin{aligned}
&\frac{\partial^2 V_k\left(\varepsilon\right)}{\partial\Theta_1^2} \\
&= \frac{1}{N^2}\frac{\partial}{\partial\Theta_1}\sum_{i,j=1}^{N}\frac{\partial}{\partial\Theta_1}G_{\sqrt{2}\Sigma}\left(\varepsilon_{ij,k}\right) \\
&= \frac{1}{N^2}\frac{\partial}{\partial\Theta_1}\sum_{i,j=1}^{N}\frac{\partial G_{\sqrt{2}\Sigma}\left(\varepsilon_{ij,k}\right)}{\partial\varepsilon_{ij,k}}\frac{\partial\varepsilon_{ij,k}}{\partial\Theta_1} \\
&= -\frac{1}{N^2}\left(\sqrt{2}\Sigma\right)^{-1}\frac{\partial}{\partial\Theta_1}\sum_{i,j=1}^{N}G_{\sqrt{2}\Sigma}\left(\varepsilon_{ij,k}\right)\times\varepsilon_{ij,k}\varepsilon_{ij,k-1}^T \\
&= -\frac{1}{N^2}\left(\sqrt{2}\Sigma\right)^{-1}\sum_{i,j=1}^{N}G_{\sqrt{2}\Sigma}\left(\varepsilon_{ij,k}\right) \\
&\quad\times\left(\varepsilon_{ij,k-2} + \varepsilon_{ij,k-1}\left(I - \left(\sqrt{2}\Sigma\right)^{-1}\varepsilon_{ij,k}^T\varepsilon_{ij,k}\right)\varepsilon_{ij,k-1}^T\right)
\end{aligned} \tag{55}$$

Note that ε_k is bounded by the proposed control algorithm, there exist two real positive number M_0 and N_0, such that

$$\varepsilon_{ij,k-2} \leq \varepsilon_{ij,k-1}M_0\varepsilon_{ij,k-1}^T \tag{56a}$$

$$\varepsilon_{ij,k}^T\varepsilon_{ij,k} \leq N_0 \tag{56b}$$

As a result,

$$\frac{\partial^2 V_k\left(\varepsilon\right)}{\partial\Theta_1^2} \leq 0 \tag{57}$$

when the following inequality holds.

$$I + M_0 - \left(\sqrt{2}\Sigma\right)^{-1}N_0 \geq 0 \tag{58}$$

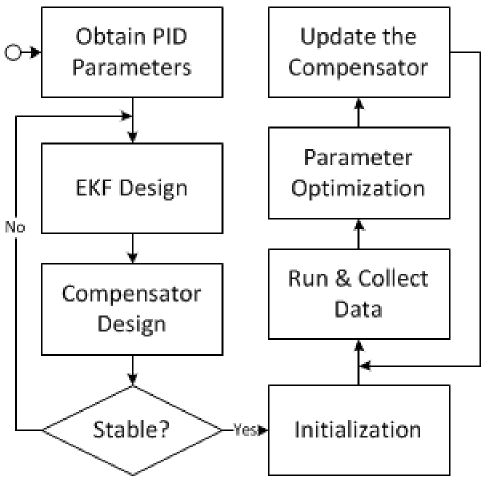


FIGURE 3. The design procedure of the presented control scheme .

It is shown that the eigenvalues of $\frac{\partial^2 V_k(\varepsilon)}{\partial \Theta^2}$ approach 0^- as $\lambda_{\min}(\Sigma)$ goes to infinity. Based on the Lemma 3 in [27], $V_k(\varepsilon)$ will be concave since $\lambda_{\min}(\Sigma)$ is sufficiently large. Moreover, the entropy function (52) is convex and has a global optimum. Similarly, the result also holds for the parameter Θ_2, which completes this proof. ∎

Therefore, the selection of design parameters can be transformed to a constrained convex optimisation problem which can be formulated as follows:

$$J_k = \min_{\Theta_1,\Theta_2} \frac{1}{N^2} \sum_{i,j=1}^{N} G_{\sqrt{2}\Sigma}\left(\varepsilon_{i,k} - \varepsilon_{j,k}\right) \tag{59a}$$

$$s.t.\ \|\Theta_1 + \Theta_2\| \le \Theta_0 \tag{59b}$$

B. THE DESIGN PROCEDURE OF THE PRESENTED CONTROL SCHEME

As we have presented the control scheme and the optimisation of the design parameters for performance enhancement. The design procedure is summarised by the following flow chart.

Remark 19: When the convex property of the target function cannot be satisfied, the intelligent optimisation can be considered to be an effective approach of the optimisation. In practice, the sub-optimum is acceptable if the stability of the closed-loop systems can be guaranteed.

Remark 20: In particular, the optimal system reference signal y^* is unknown for most of the industrial processes. In this case, the optimisation approach should be applied before the control design, and the optimal reference can be predicted by operational optimal control technology.

VI. SIMULATION EXAMPLES

Without loss of generality, consider a typical discrete-time SISO system which is represented by the following equations:

$$x_{k+1} = \begin{bmatrix} 0 & 1 \\ -0.5 & -0.6 \end{bmatrix} x_k + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_k + 0.3 u_k \sin x_k + w_k$$

$$y = \begin{bmatrix} 1 & 1 \end{bmatrix} x_k + v_k$$

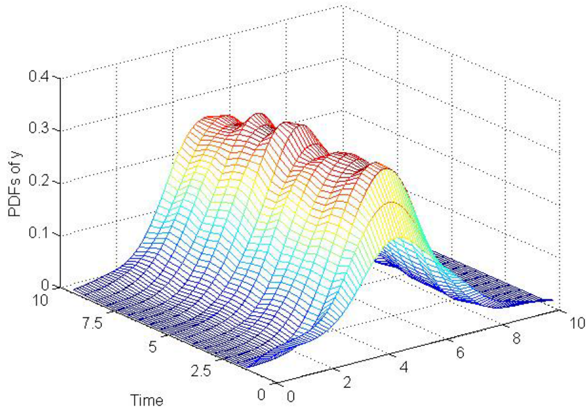


FIGURE 4. The PDFs of the system output y.

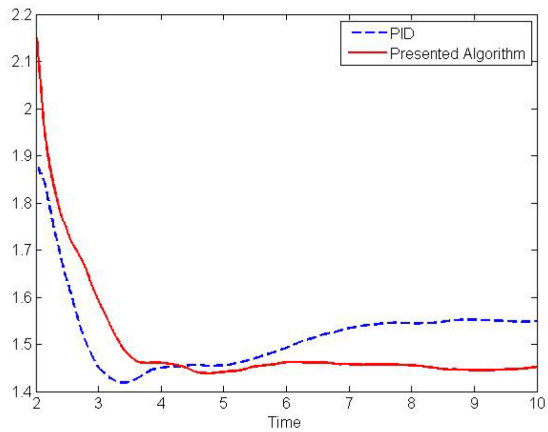


FIGURE 5. The expectation entropy curves of the system tracking error.

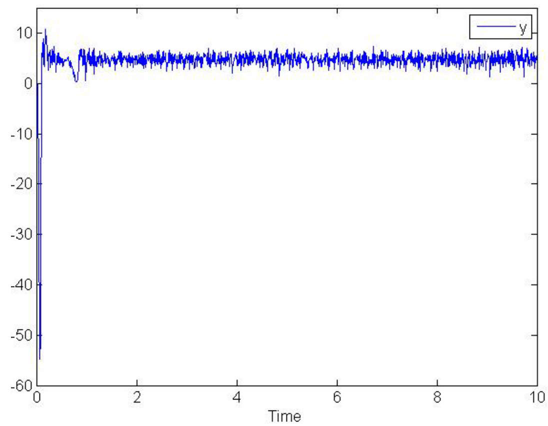


FIGURE 6. The system output.

where the sampling time is 0.01s and the noises obey the Gaussian distribution as follows:

$$v_k, \quad w_k \sim N(0, 0.2)$$

Following the design procedures, the results are given by Figures 4-10 since the reference value is chosen as 5. The probability density functions of the outputs of this system are

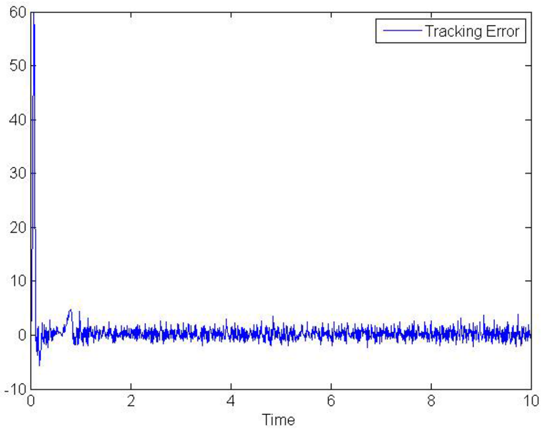


FIGURE 7. The tracking error of the system output.

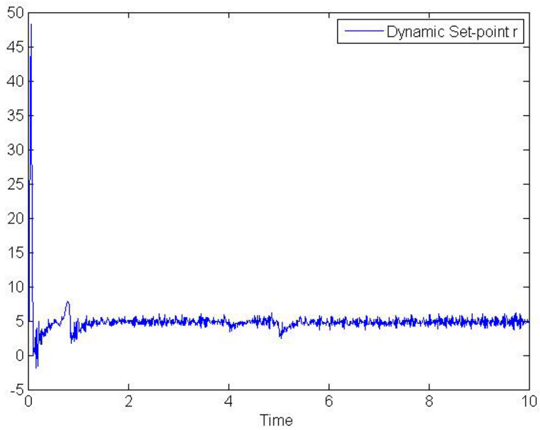


FIGURE 8. The dynamic set-point r.

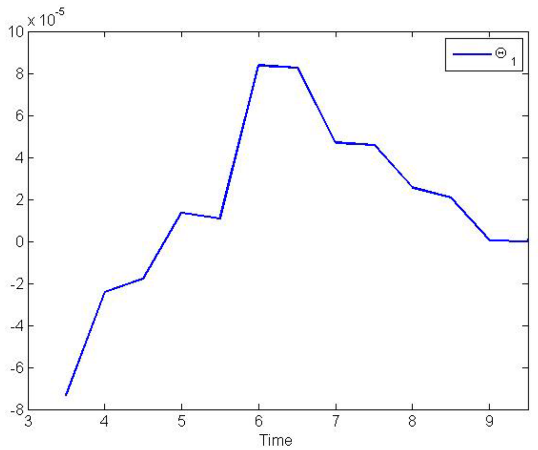


FIGURE 9. The design parameter Θ_1 of the presented control algorithm.

shown in Figure 4. Comparing with the standard PID controller, the expectation values of the systems error's entropy is given by Figure 5 while the presented control algorithm can provide the enhancement of the performance and the randomness of the system is attenuated. For this numerical example,

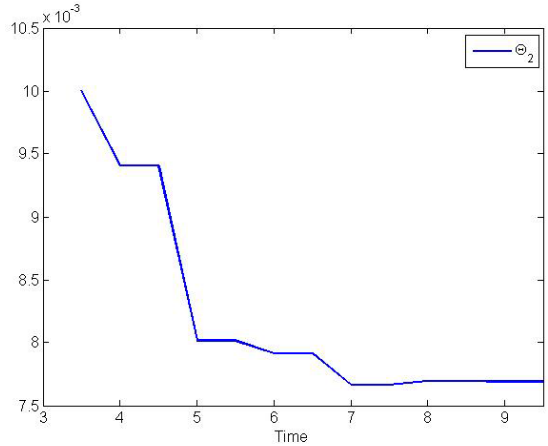


FIGURE 10. The design parameter Θ_2 of the presented control algorithm.

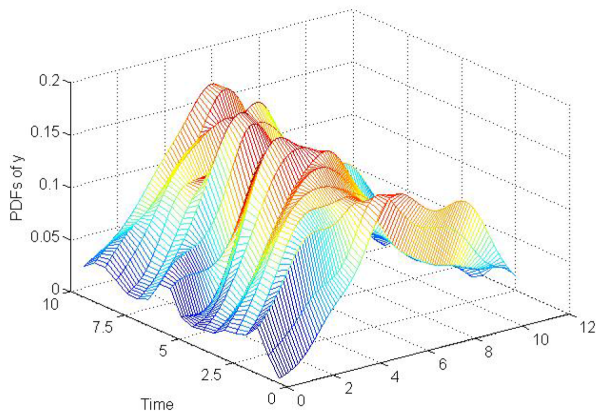


FIGURE 11. The PDFs of the system output y subjected to Gamma noises.

the PID parameters are selected as $K_p = 0.5$, $K_i = 0.01$ and $K_d = 0$, then the dynamic set-point is calculated based on these parameters. Moreover, Figure 6,7 and 8 illustrate the curves of system output, the system tracking error and the dynamic set-point signal which imply that the system is stable using the presented control algorithm. Furthermore, the parameters Θ_1 and Θ_2 are optimised by gradient descent optimisation, and the searching paths are shown in Figure 9 and Figure 10.

In practice, the noises cannot be assumed as Gaussian noises only, therefore the non-Gaussian noises should be considered as well. Although the theoretical analysis in this paper is given subjected to Gaussian distribution, we can still validate the robustness and resilience of the algorithm using the non-Gaussian noise. In particular, for this numerical example, if the noises obey Gamma distribution as follows:

$$v_k, \quad w_k \sim Gamma\,(5, 1)$$

The system can be stabilised by the presented control algorithm and the associated results are given by the figures 11 and 12. Figure 11 shows the PDFs of the system outputs which are subjected to the Gamma noises and the

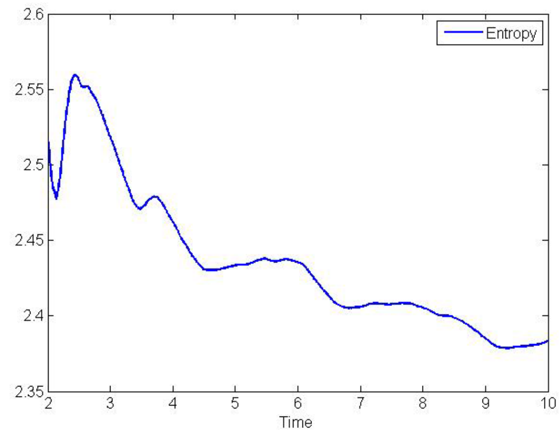


FIGURE 12. **The expectation entropy curves of the system tracking error subjected to Gamma noises.**

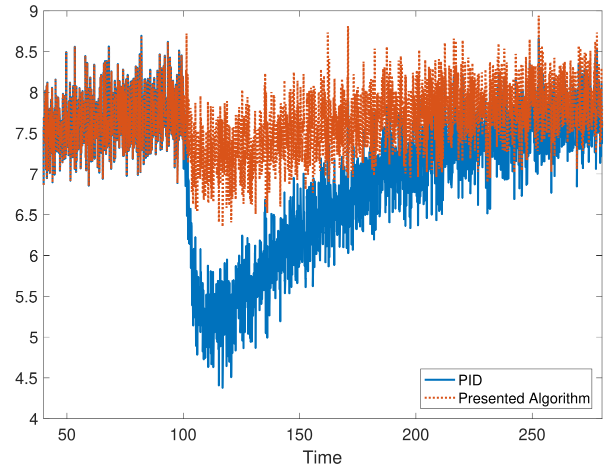


FIGURE 14. **The comprised performances of the twin-tank level process with external disturbance.**

expectation of the tracking error's entropy is given using Figure 12. Notice that the PDF in Fig. 11 becomes shaper along the decreasing of the entropy.

The power of the presented algorithm can be demonstrated for practical system as well. Thus another practical example is given based on the twin-tank level process system whose structure is shown by Figure 13.

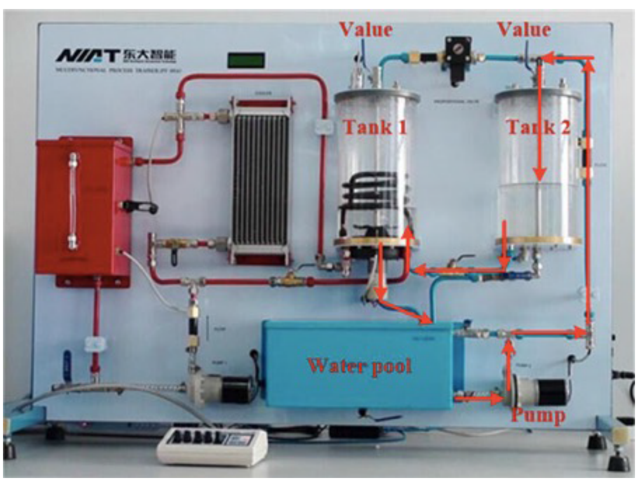


FIGURE 13. **The structure of the twin-tank level process system.**

The model and the parameters with instruction is setup in [15]. In particular, the system can be formulated as follows:

$$\begin{aligned} x_{1,k+1} &= -\frac{h}{A_1}\left(c_1 + k_1\sqrt{x_{1,k}} - k_0\sqrt{x_{2,k} - x_{1,k}}\right) \\ &\quad + x_{1,k} + w_{1,k} \\ x_{2,k+1} &= \frac{h}{A_2}\left(k_4 u_{2,k} - c_2 - k_0\sqrt{x_{2,k} - x_{1,k}}\right) \\ &\quad + x_{2,k} + w_{2,k} \\ y_k &= x_{1,k} + v_k \end{aligned} \tag{60}$$

where x_1 and x_2 stand for the levels of tank 1 and tank 2. A_1 and A_2 are the cross-sectional area. c_1 and c_2 are constant parameters of the valve and pump. k_0, k_1 and k_4 denote the ratio of the valves. h is the sampling time. In addition, w and v are the process noise and measurement noise, respectively.

In particular, $A_1 = A_2 = 167.4\, cm^2$ while $c_1 = 0$ and $c_2 = 2.88$. k_0, k_1 and k_4 are equal to 0.7, 0.25 and 0.1, respectively. The control input u_2 is set as 30 while the equilibrium points are 0.23 and 0.26 for both tanks. Moreover, w_k and v_k is subjected to Gaussian distribution with zero means and 0.1 variance. The PI controller can be given as $Kp = 75.5$ and $Ki = 0.07$. Suppose that the actuator of the system has a fault at $100s$ the amplification of the control input is reduced to 20%, the presented algorithm will increase the set-point value which attenuate the effect of this fault. Not only for entropy attenuation, the performance comparison is given to show the benefit of the presented algorithm using Fig. 14.

VII. CONCLUSIONS

In this paper, a new control scheme has been presented to enhance the performance of a class of stochastic nonlinear systems. In particular, an additional compensation loop has been designed to add onto the PID loop using a set-point dynamic adjustment strategy, where the EKF-based full system information estimation has been adopt for enhancing the performance. In summary, it has been shown that the structure of this control scheme has been divided into two layers.

Based upon the minimum entropy performance criterion, the design parameters can be obtained following the constrained convex optimisation. The stability analysis shows the parametric guarantee of the closed-loop systems regarding to the system stability in probability one. Finally, the numerical examples are give to illustrate the effectiveness of the presented control scheme and the associated results indicate that the performances of the closed-loop systems have been enhanced in terms of randomness attenuation. Potentially, the extended scheme can be used to enhance the performance in many aspects using various performance criteria.

APPENDIX
PROOF OF LEMMA

Proof of Lemma 16: Constructing a linear deterministic system:

$$x_{k+1} = Ax_k + Bu_k \tag{A.1}$$

where x_k is the state of the system, A and B are coefficient matrices in appropriate dimensions and u_k is the control input.

Consider the performance index:

$$J = \sum_{k=1}^{\infty} \left(x_k^T Q x_k + u_k^T R u_k \right) \tag{A.2}$$

where Q and R are artificial symmetric positive definite matrices.

Note that for any matrix A, we can always find a matrix B such that the system (A, B) is controllable, then the linear quadratic regulator (LQR) can be designed as follows:

$$u_k = -Fx_k \tag{A.3a}$$

$$F = \left(R + B^T PB\right)^{-1} B^T PA \tag{A.3b}$$

$$P = A^T PA - A^T PB\left(R + B^T PB\right)^{-1} B^T PA + Q \tag{A.3c}$$

where the Riccati equation in terms of P has the stable positive definite solution.

Moreover, P can be restated by

$$P = A^T (I + \Pi) PA + Q \tag{A.4}$$

where $\Pi = -PB\left(R + B^T PB\right)^{-1} B^T$.

Using the matrix inverse lemma for Π, we have

$$\Pi = -\left(P^{-1} + BR^{-1}B^T\right)^{-1} BR^{-1}B^T = -\left[I + \left(B^{-1}\right)^T RB^{-1}P^{-1}\right]^{-1} \tag{A.5}$$

Since $\left(B^{-1}\right)^T RB^{-1}P^{-1} > 0$, there exists a real constant $a \geq 1$ such that

$$I + \Pi = \frac{1}{a} I \tag{A.6}$$

which leads to

$$P \geq A^T \left(\frac{1}{a+b} P + \left(A^{-1}\right)^T QA^{-1} \right) A \tag{A.7}$$

where $b \geq 0$ is any real constant.

We can always find a matrix Q which meets the following inequality.

$$\left(A^{-1}\right)^T QA^{-1} \geq cP, \quad 0 < c \leq 1 \tag{A.8}$$

we have

$$P \geq A^T \left(\left(\frac{1}{a+b} + c \right) P \right) A \tag{A.9}$$

Furthermore, inverse the both sides of the inequality, the following inequality is obtained.

$$P^{-1} \leq A^{-1}P^{-1}\alpha\left(A^{-1}\right)^T \tag{A.10}$$

Finally, we can have

$$AP^{-1}A^T \leq \alpha P^{-1} \tag{A.11}$$

Note that there exists a constant $c > 1$ which meets the inequality (A.8), if $\|A\| < 1$, which leads to the existing of α, where $0 < \alpha < 1$. ■

Proof of Lemma 17: Since s_k is a vector with two elements, we have

$$s_k \leq \|s_k\| = \left\| \left(g_k(x_k, u_k) - g_k(\hat{x}_k, u_k) \right) \right\| + \|(\Pi_3 \tilde{x}_{k-1} + C_d E_d g_k(x_k, u_k))\| \tag{A.12}$$

Based upon the assumptions for nonlinear terms, the following inequality can be obtained.

$$s_k \leq L_1 \|\tilde{x}_k\| + \|\Pi_3\| \|\tilde{x}_{k-1}\| + L_2 \|C_d E_d\| \|x_k\| \tag{A.13}$$

Using the assumptions for the states and the estimated errors, we have

$$\|s_k\| \leq M_1 \|\tilde{x}_k\| + M_2 \|\varepsilon_k\| + M_3 \tag{A.14}$$

Note that $\|\tilde{x}_k\| \leq \|\zeta_k\|$ and $\|\varepsilon_k\| \leq \|\zeta_k\|$, s_k can be further expressed by

$$\|s_k\| \leq (M_1 + M_2) \|\zeta_k\| + M_3 \tag{A.15}$$

Therefore, the following inequality can be obtained.

$$s_k^T P^{-1} \left(2A_f \zeta_k + s_k\right) \leq \left\| s_k^T P^{-1} \left(2A_f \zeta_k + s_k\right) \right\| \leq \left\| P^{-1} \right\| \left(N_1 \|\zeta_k\|^2 + N_2 \|\zeta_k\| + M_3^2 \right) \tag{A.16}$$

where $N_1 = (M_1 + M_2)\left(2\left\|A_f\right\| + M_1 + M_2\right)$ and $N_2 = 2M_3\left(\left\|A_f\right\| + M_1 + M_2\right)$.

Suppose that P is real positive definite matrix, thus $P \geq \underline{p}I$. In other words, we have

$$\left\| P^{-1} \right\| \leq \frac{1}{\underline{p}} \tag{A.17}$$

which leads to the first inequality.

Similarly, notice that

$$d_k \leq (\|G\| + \|C_d G_d\|) \|w_k\| + \left(\left\|K_f D\right\| + \|D_d\|\right) \|v_k\| \tag{A.18}$$

Moreover, we have

$$E\{\|d_k\|\} \leq \delta \tag{A.19}$$

Then the following inequality is given.

$$E\left\{d_k^T P^{-1} d_k\right\} \leq \frac{1}{\underline{p}} E\left\{\|d_k\|^2\right\} \tag{A.20}$$

Thus the proof is completed using inequality (A.17). ■

REFERENCES

[1] K. J. Åström and T. Hägglund, *Advanced PID Control*. Research Triangle, NC, USA: ISA-The Instrumentation, Systems and Automation Society, 2006.

[2] T. Chai, "Optimal operational control for complex industrial processes," *IFAC Proc. Volumes*, vol. 45, no. 15, pp. 722–731, 2012.

[3] P. Zhou, T. Chai, and H. Wang, "Intelligent optimal-setting control for grinding circuits of mineral processing process," *IEEE Trans. Autom. Sci. Eng.*, vol. 6, no. 4, pp. 730–743, Oct. 2009.

[4] T. Chai, J. Ding, and F. Wu, "Hybrid intelligent control for optimal operation of shaft furnace roasting process," *Control Eng. Pract.*, vol. 19, no. 3, pp. 264–275, Mar. 2011.

[5] S. Skogestad and I. Postlethwaite, *Multivariable Feedback Control: Analysis and Design*, vol. 2. New York, NY, USA: Wiley, 2007.

[6] R. E. Kalman, "A new approach to linear filtering and prediction problems," *J. Basic Eng.*, vol. 82, no. 1, pp. 35–45, Mar. 1960.

[7] S. S. Haykin, *Kalman Filtering and Neural Networks*. Hoboken, NJ, USA: Wiley, 2001.

[8] S. J. Julier and J. K. Uhlmann, "Unscented filtering and nonlinear estimation," *Proc. IEEE*, vol. 92, no. 3, pp. 401–422, Mar. 2004.

[9] R. Van der Merwe and E. A. Wan, "The square-root unscented Kalman filter for state and parameter-estimation," in *Proc. IEEE Int. Conf. Acoust., Speech, Signal Process.*, vol. 6, May 2001, pp. 3461–3464.

[10] F. Gustafsson, "Particle filter theory and practice with positioning applications," *IEEE Aerosp. Electron. Syst. Mag.*, vol. 25, no. 7, pp. 53–82, Jul. 2010.

[11] X. Yin, Q. Zhang, H. Wang, and Z. Ding, "RBFNN-based minimum entropy filtering for a class of stochastic nonlinear systems," *IEEE Trans. Autom. Control*, vol. 65, no. 1, pp. 376–381, Jan. 2020.

[12] G.-H. Yang and W.-W. Che, "Non-fragile H_∞ filter design for linear continuous-time systems," *Automatica*, vol. 44, no. 11, pp. 2849–2856, Nov. 2008.

[13] Q. Zhang and L. Hu, "Probabilistic decoupling control for stochastic nonlinear systems using EKF-based dynamic set-point adjustment," in *Proc. UKACC 12th Int. Conf. Control (CONTROL)*, Sep. 2018, pp. 330–335.

[14] B. W. Silverman, *Density Estimation for Statistics and Data Analysis*, vol. 26. Boca Raton, FL, USA: CRC Press, 1986.

[15] Y. Zhou, Q. Zhang, H. Wang, P. Zhou, and T. Chai, "EKF-based enhanced performance controller design for nonlinear stochastic systems," *IEEE Trans. Autom. Control*, vol. 63, no. 4, pp. 1155–1162, Apr. 2018.

[16] Q. Zhang, J. Zhou, H. Wang, and T. Chai, "Minimized coupling in probability sense for a class of multivariate dynamic stochastic control systems," in *Proc. 54th IEEE Conf. Decis. Control (CDC)*, Dec. 2015, pp. 1846–1851.

[17] Q. Zhang, A. Wang, and H. Wang, "Minimum mutual information control for multi-variable non-Gaussian stochastic systems," in *Proc. Int. Conf. Adv. Mech. Syst. (ICAMechS)*, Aug. 2015, pp. 497–502.

[18] G. C. Goodwin and K. S. Sin, *Adaptive Filtering Prediction and Control*. Chelmsford, MA, USA: Courier Corporation, 2014.

[19] R. G. Agniel and E. I. Jury, "Almost sure boundedness of randomly sampled systems," *SIAM J. Control*, vol. 9, no. 3, pp. 372–384, Aug. 1971.

[20] T.-J. Tarn and Y. Rasis, "Observers for nonlinear stochastic systems," *IEEE Trans. Autom. Control*, vol. 21, no. 4, pp. 441–448, Aug. 1976.

[21] K. Reif, S. Gunther, E. Yaz, and R. Unbehauen, "Stochastic stability of the discrete-time extended Kalman filter," *IEEE Trans. Autom. Control*, vol. 44, no. 4, pp. 714–728, Apr. 1999.

[22] E. Parzen, *Stochastic Processes*, vol. 24. Philadelphia, PA, USA: SIAM, 1999.

[23] L. Paninski, "Estimation of entropy and mutual information," *Neural Comput.*, vol. 15, no. 6, pp. 1191–1253, Jun. 2003.

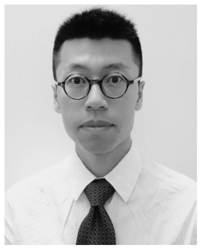
[24] Y. Jiang, J. Fan, T. Chai, and T. Chen, "Setpoint dynamic compensation via output feedback control with network induced time delays," in *Proc. Amer. Control Conf. (ACC)*, Jul. 2015, pp. 5384–5389.

[25] H. Wang, "Minimum entropy control of non-Gaussian dynamic stochastic systems," *IEEE Trans. Autom. Control*, vol. 47, no. 2, pp. 398–403, Feb. 2002.

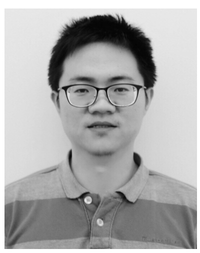
[26] T. M. Cover and J. A. Thomas, *Elements of Information Theory*. Hoboken, NJ, USA: Wiley, 2012.

[27] Y. Liu, H. Wang, and C. Hou, "UKF based nonlinear filtering using minimum entropy criterion," *IEEE Trans. Signal Process.*, vol. 61, no. 20, pp. 4988–4999, Oct. 2013.

XIAFEI TANG received the B.Eng. in electrical engineering and automation from Beihang University, in 2006, and the Ph.D. degree in electrical and electronic engineering from the University of Manchester, U.K., in 2012. After that, she joined the Changsha University of Science and Technology, in 2013. She is currently working as a Lecturer with the School of Electrical and Information Engineering. Her research interests including disturbance rejection, nonlinear systems control, the stability analysis of power systems, and risk assessment of power systems.



QICHUN ZHANG (Senior Member, IEEE) received the B.Eng. degree in automation and the M.Sc. degree in control theory and control engineering from Northeastern University, China, in 2008, and 2010, respectively, and the Ph.D. degree in electrical and electronic engineering from the University of Manchester, U.K., in 2016. Before joining Bradford, he was a Senior Lecturer in dynamics and control with De Montfort University, a Senior Research Officer in neural engineering with the University of Essex, and an Academic Visitor with the Control Systems Centre, University of Manchester. He is currently a Lecturer in computer science with the University of Bradford, U.K. His research interests include stochastic dynamic systems, probabilistic coupling analysis, decoupling control, performance optimization, brain–computer interface, and computational modeling for peripheral nervous systems. He serves more than 20 international journals as an Active Reviewer. He is also an Associate Editor of IEEE ACCESS and the *Journal of Intelligent Manufacturing*. He is also an Academic Editor of *PLoS ONE* and an Editorial Board Member of other two journals.



LIANG HU (Member, IEEE) received the B.E. and M.E. degrees from the Harbin Institute of Technology, China, in 2008 and 2010, respectively, and the Ph.D. degree from Brunel University London, U.K., in 2016. He was a Lecturer with the School of Computer Science and Informatics, De Montfort University, U.K. He did the Postdoctoral Research at Queen's University Belfast and then Loughborough University, from 2016 to 2018. He is currently a Lecturer with the School of Computer Science and Electronic Engineering, University of Essex, U.K. His research interests include signal processing, control and decision and their applications in autonomous systems, and intelligent transport systems.